Derivatives and Shapes of Curves (Section 4.3)

Intro

Previously we looked at the graphs of $f(x)$, $f'(x)$, and $f''(x)$ to get information about the function. Today we will do the same thing but instead work from equations.

Overview

Today we will cover:

- Shape of a graph
- Sketching graphs

Shape of a graph

The following information will help determing the shape of the graph of $f(x)$:

- Intervals where $f(x)$ is increasing or decreasing
- Local maxs/mins
- Inflection points
- Intervals where $f(x)$ is concave up/down

Extended example

We will do an extended example with

$$f(x) = 2x^3 + 3x^2 - 36x + 5$$

Intervals where $f(x)$ is increasing/decreasing

Find the derivative of $f(x)$

Find critical points + undefined points

Draw number line and plot these points

Plug x-values in between these points into $f'(x)$ to determine slopes

Maxs/mins

Examine the number line from the previous step

Determine if the critical points are maxs/mins/neither

Calculate corresponding y-values

Inflection points

Find $f''(x)$ (take the derivative of $f'(x)$)

Set it equal to 0, solve

The inflection points are the roots where $f''(x)$ changes sign

Concave up/Concave down

Draw a number line and plot inflection points

Plug x-values in between inflection points to determine concavity

Example

Calculate the 4 aspects for the function. Use this information to sketch the function.

$$f(x) = \frac{30x}{x^2 + 9}$$

Short Survey

One thing we do that I would like to **keep**

One thing we do that I'd like to **stop/change**

One thing we don't do that I would like to **start**

Example

Calculate the 4 aspects for the function. Use this information to sketch the function.

$$f(x) = 10x^2 \ln(x)$$