Local and Absolute Extrema (Section 4.2)

Intro

Today we'll talk about how to find the highest and lowest points on a graph.

- Finding local extrema
- Finding absolute extrema
- Information from the second derivative

Recall

When we say local max/min, we mean a **point** (x,y)

When we say **value** of a max/min, we mean the *y*-value

Critical points

A **critical point** is a point where either

$f'(x) = 0$, OR

$f(x)$ exists, but $f'(x)$ does not exist

These are points where local extremum may (but do not have to) appear.

Finding local maxs/mins

Find the derivative of $f(x)$

Find the critical points of $f(x)$. Usually, this just means solving for x such that $f'(x) = 0$.

Draw a chart showing where $f'(x)$ is positive or negative.

Use this to determine which points are maxs/mins

Example

Calculate the local maxs/mins of $f(x)$.

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 6x + 1$$

Example

Calculate the local maxs/mins of $f(x)$.

$$f(x) = x^{2/5} + 4x^{-3/5}$$

Warning

According to the definition in your book, local max/mins never occur at the edge of the graph.

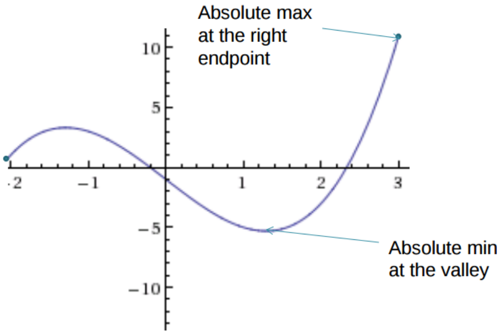
Absolute Maxs/Mins

Before we only cared about local maxs/mins, not the whole graph

Absolute maxs/mins only care about the highest/lowest points on the whole graph

We will only compute maxs/mins on closed intervals (e.g. $[-3, 2]$)

Example



Absolute max
at the right
endpoint
10
5
-2
–1
1
2
3
–5
–10
Absolute min
at the valley

Absolute Maxs/Mins

Absolute maxs/mins must be either critical points or end points

So we jsut find each of these x-values, then compare the y-values to see which is biggest/smallest

Possible questions: location of maxs/mins (x-values) or value of max/min (y-value)

Finding Absolute Extrema

Find the derivative of $f(x)$

Find the critical points

Make a table of x-values and y-values containing all critical points and end points

Select the points with the largest/smallest y-values

Example

Find the absolute maximum and minimum for $f(x)$ on the interval $[-2, 3]$.

$$f(x) = 2x^3 + 2x^2 - 2x + 1$$

Example

Find the absolute maximum and minimum values for $f(x)$ on the interval $[0, 5]$.

$$f(x) = \sin\left(\frac{\pi}{4}(x+1)\right)$$

Example

Find the absolute maximum and minimum values for $f(x)$ on the interval $[-1, 2]$.

$$f(x) = x^4 - 2x^2$$

Extrema and Second Derivatives

The second derivative can help us determine if a critical point is a max, a min, or neither.

- If $f'(a) = 0$ and $f''(a) > 0$ then the graph is a "smiley" at a and thus there is a min at $x = a$
- if $f'(a) = 0$ and $f''(a) < 0$ then the graph is a "frowny" at a and thus there is a max at $x = a$
- If $f'(a) = 0$ and $f''(a) = 0$ then we cannot determine if it is a max/min/neither from the second derivative
- If $f'(a) \neq 0$ then there cannot be a max/min at $x = a$ no matter what $f''(a)$ is.

Example

Suppose that $f(x)$ is a function where

$$f'(a) = 0,\ f''(a) < 0$$

$$f'(b) > 0,\ f''b = 0$$

Classify the points at a, b as local maxs, mins, or neither.