Derivatives of Log Functions, Applications in Natural and Social Sciences (Sections 3.7 and 3.8)

Intro

The lecture today goes over the last set of derivative rules we need for this class. We will cover logarithmic and (more) exponential derivative rules. Aftwerwards, we'll talk about some applications.

Exponential Rule

For any number a:
If

$$f(x) = a^x,$$

then

$$f'(x) = \ln(a)a^x$$

Example

Find the derivative.

$$f(x) = 5^{\sin(x)}$$

Example

Find the derivative.

$$f(x) = \cot(8^x)$$

Log Rule

New derivative rule:

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

Note: this rule only works for the natural log (ln).

Example

Find the derivative.

$$f(x) = \frac{\ln(x)}{x^5}$$

Example

Find the derivative.

$$f(x) = \ln(x^2 + \sqrt{x})$$

Example

Find the derivative.

$$f(x) = \cos(\ln(x^3 - e^x))$$

Example

Find the derivative.

$$f(x) = \ln\left(e^{2x} + \frac{x}{5+x}\right)$$

Example

Find the derivative.

$$f(x) = \ln\left(\frac{x^3 e^x (x^2+1)}{\sin(x)}\right)$$

Hint: simply f first.

Log base a

For a base a logarithm:

$$f(x) = \log_a(x)$$

$$f'(x) = \frac{1}{x \cdot \ln(a)}$$

Example

Find the derivative.

$$f(x) = \log_{10}(x^{3/2} + \sin(x))$$

Example

The formula for calculating volume in Watts from decibel Watt units is

$$P(x) = 10^{x/10}$$

For example, 20 dBW corresponds to a volume of

$$P(20) = 10^{20/10} = 100 \text{ W}$$

What is the rate of change of the volume in W with respect to dBW at 30 dBW?

Psychology

Psychologists model the spread of a rumor through a population as

$$p(t) = \frac{1}{1 + 2e^{-t/5}}$$

where $p(t)$ is the fraction of the population who know the rumor at time t days.

Psychology

Model:

$$p(t) = \frac{1}{1 + 2e^{-t/5}}$$

After how many days with 90% of the population know the rumor?

How fast is the rumor spreading at that point?

Health

Body temperature fluctuates throughout the day. It is usually highest in the afternoon and lowest in the early morning (while asleep). It can be modelled approximately by a trigonometric function

$$f(t) = 98.5 - 1.1\cos\left(\frac{\pi}{12}(t-4)\right),$$

where t is the number of hours after midnight.

Health

Model:

$$f(t) = 98.5 - 1.1\cos\left(\frac{\pi}{12}(t-4)\right)$$

What are the local maxes/mins for body temperature?

Kinesiology

Weight lifters value smooth, controlled motion for weightlifting. This means if we track the motion of the weight, we want to avoid areas where there is a sudden spike in the velocity of the lift.
Suppose a bench press lift is modeled by

$$h(t) = t^4 - 8t^3 + 23t^2 - 28t + 14$$

where t is in seconds and h is the height in inches.

Kinesiology

Model:

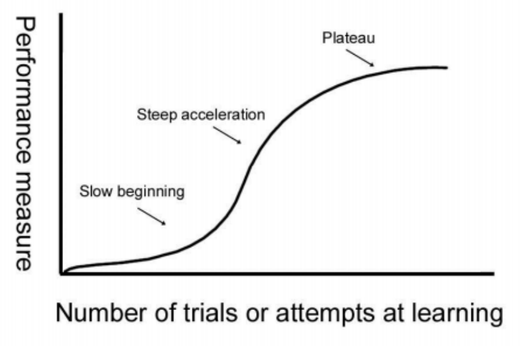
$$h(t) = t^4 - 8t^3 + 23t^2 - 28t + 14$$

Find the values where $h'(t)$ has a local max/min.

How is the weight lifter doing?

Education

The learning curve represents how a student learns material based on time.



Education

What does the derivative of the learning curve represent?

When should the derivative be smallest? Largest?

What is wrong with the graph on the previous slide?

Biomedical Science

You perform an experiment using millions of bacterial microbes. A specialized laser is used to measure the success of the experiment. The data is then entered into a computer and the computer processes the data. The time it takes to process the data is:

$$t(s) = \ln(5s^2 + 3s - 1),$$

where s is the number of microbes and t is the time in minutes

Biomedical Science

Model:

$$t(s) = \ln(5s^2 + 3s - 1)$$

If you need to complete the computation in 1 hours, what is the maximum number of microbes you can use for the experiment?

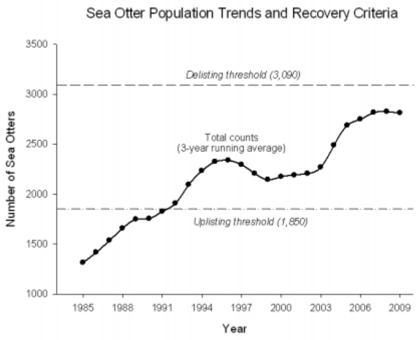
What is the rate of change of the computation time if $s = 10^{10}$?

Sports Management

You manage the finances for a minor league baseball team (Brazos Bombers). You are tasked with setting ticket prices to maximize the team's profits. After factoring in the costs, you find the profit equation:

$$p(x) = -10x^2 + 400x - 1000$$

Animal Science



In what years was there a local max of the sea otter population?
Does this have implications for conservation efforts?

If time permits

More group problems?

$$f(x) = (\ln(1+e^x))^2$$

$$f(x) = \sec\left(\ln\left(\frac{x^2+1}{x-e^x}\right)\right)$$