Derivatives and Rates of Change (Section 2.6)

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Outline

Concept of derivative

Word problems

Math examples

Derivative = Rate of change

The **derivative** of a function is the instantaneous rate of change of the function.

Derivative of position = velocity

Derivative of volume in tank = rate of drainage

Derivative of mass of a type of molecule = rate of reaction

Derivative = Slope of graph

The derivative at $x = a$ is also the slope of the graph at a given $x = a$.

Formal definition of derivative

The derivative of $f(x)$ at $x = a$ is defined as:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Slopes of secant lines vs derivatives

	Slope of secant line	Derivative
Physical meaning	Average velocity	Instantaneous velocity
Formula	$\frac{y_2-y_1}{x_2-x_1} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$	$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

Example

Alfred runs a 100 meter dash in 10 seconds. The equation for his position during the race is $p(t) = t^2$.

Compute his average velocity over the whole race.

Compute his instantaneous velocity at $t = 2$ and $t = 8$

How do these compare?

Draw the graph of the derivative from a graph

Derivative - Easier to use formula

Instead of

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

, we can also use

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Example

Calculate the derivative of the function at $x = 2$

$$f(x) = 3x^2 - 5x$$

Example

Calculate the derivative of the function at $x = -1$

$$f(x) = \frac{5x}{4 - x}$$

Example

Calculate the derivative of the function at $x = 8$

$$f(x) = \sqrt{2x}$$

Example

Calculate the derivative of the function at $x = 1$

$$f(x) = 3x^2 - 6x + 4$$

Tangent Lines

To find the tangent line at a point (x_0, y_0),

Calculate the derivative at $x = x_0$.

Use point-slope form with the slope $m = f'(x_0)$

Example

Calculate the equation of the tangent line to the curve $f(x)$ at the point $(2, -1)$.

$$f(x) = \frac{1}{1-x}$$

Example

Find the equation of the tangent line to the curve $f(x)$ at the point $(9, 2)$.

$$f(x) = \sqrt{x - 5}$$

Going backwards

The following limit is the derivative for some function $f(x)$ at $x = a$ What could $f(x)$ and a be?

$$\lim_{h \to 0} \frac{\sin\left(h + \frac{\pi}{2}\right) - 1}{h}$$