

# Continuity (Section 2.4)

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9:35 - 10:50 AM

# Outline

Definition of continuous

Discontinuity examples

One sided continuity

Harder examples

## Intuitive idea

A **continuous function** is a function that doesn't have any gaps, jumps, or holes. (You can draw its graph without taking your pencil off the paper.)

# Continuity at a point

Idea: Look at tiny neighborhood around the point to determine continuity of the function at the point.

## Definition using limits (IMPORTANT)

The function  $f(x)$  is continuous at  $x = a$  if

The limit  $\lim_{x \rightarrow a} f(x)$  exists.

The function  $f(x)$  is defined at  $x = a$

The two match:  $\lim_{x \rightarrow a} f(x) = f(a)$ .

# First requirement: $\lim_{x \rightarrow a} f(x)$ exists

This means that

The limit from the left  $\lim_{x \rightarrow a^-} f(x)$  exists,

the limit from the right  $\lim_{x \rightarrow a^+} f(x)$  exists,

and they are both equal  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

## Second requirement: $f(x)$ is defined at $x = a$

This means that the number  $a$  is in the domain of  $f(x)$ .

Recall: Possible issues include dividing by 0, even roots of negative numbers, and log's of nonpositive numbers.

Third requirement:  $\lim_{x \rightarrow a} f(x) = f(a)$

Calculate both sides, see if they agree.

Usually only piecewise functions violate this rule.



## Continuity on an interval

If a function  $f(x)$  is continuous at every point in the interval  $(a, b)$ , then we say that  $f(x)$  is continuous on  $(a, b)$ .

# Discontinuities

Removable discontinuities

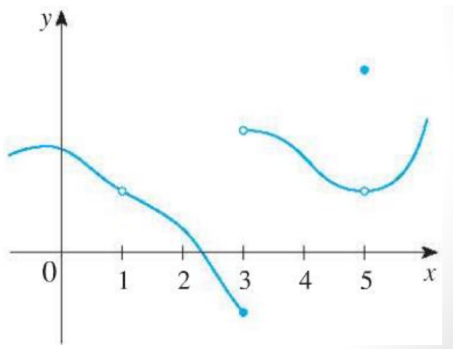
Jump discontinuities

Infinite discontinuities

Other types

Which of the three rules does  $f(x)$  break at:

1.  $x = 1$
2.  $x = 3$
3.  $x = 4$
4.  $x = 5$



## Example

Identify the discontinuities of the following function:

$$f(x) = \frac{x^2 + 6x + 5}{x^2 - 2x - 3}$$

## Example

Identify the discontinuities of the following function:

$$f(x) = e^{1/x} + \frac{x^2 - 1}{x + 1}$$

# One sided continuity

Take the definition of continuity, and replace each limit with a one-sided limit.

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# One sided continuity

The function  $f(x)$  is continuous at  $x = a$  **from the left** if

The limit  $\lim_{x \rightarrow a^-} f(x)$  exists.

The function  $f(x)$  is defined at  $x = a$

The two match:  $\lim_{x \rightarrow a^-} f(x) = f(a)$ .



# One sided continuity

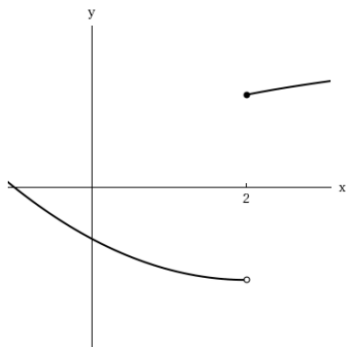
The function  $f(x)$  is continuous at  $x = a$  **from the right** if

The limit  $\lim_{x \rightarrow a^+} f(x)$  exists.

The function  $f(x)$  is defined at  $x = a$

The two match:  $\lim_{x \rightarrow a^+} f(x) = f(a)$ .

Is  $f(x)$  continuous from the left/right at  $x = 2$ ?



## Harder example

Find the number  $c$  such that  $f(x)$  is continuous at  $x = 3$ .

$$f(x) = \begin{cases} cx^2 + 6x, & x < 3 \\ x^3 - cx, & x \geq 3 \end{cases}$$

## Harder example

Find the number  $c$  such that  $f(x)$  is continuous at  $x = -1$ .

$$f(x) = \begin{cases} x^3 - 2x^2 + cx, & x < -1 \\ cx^2 + 7x, & x \geq -1 \end{cases}$$