Calculating Limits (Section 2.3)

Feb 2, 2017
9:35 - 10:50 AM

Outline

Limits of continuous functions – Plug it in

Cancelling fractions

Rationalizing fractions

Limits of piecewise functions

Plug it in

If there is nothing weird going on, then we can evaluate the limit by plugging the number in.

Example:

$$\lim_{x \to 2} x^2 + 1$$

Example:

$$\lim_{x \to 0} e^{2x} + 3x - 2$$

Plug it in

"Nothing weird going on" means that the function is continuous at the limit point.
In particular, we are not dividing by 0, plugging in negative numbers to square roots, or taking logs of negative numbers.

Limit Laws

$$\lim_{x \to a} f(x) + g(x) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$$

$$\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

Limit Laws

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \text{ (as long as you aren't dividing by 0)}$$

$$\lim_{x \to a} (f(x))^n = \left(\lim_{x \to a} f(x) \right)^n$$

More generally, you can take limits inside of continuous functions.

Cancelling to get limit

If you get $\frac{0}{0}$ when trying to evaluate the limit of a fraction, you can try to factor and cancel.

Example:

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

Example

Calculate the limit of $f(x)$ as $x \to -1$.

$$f(x) = \frac{x^2 - 2x - 3}{x + 1}$$

Example

Calculate the limit

$$\lim_{x \to 5} \frac{x^2 - 2x - 15}{x^2 - 6x + 5}$$

Rationalizing the Numerator

Example:

$$\lim_{h \to 0} \frac{\sqrt{25+h}-5}{h}$$

Rationalizing the Numerator

Technique:

$$\frac{\sqrt{a}-b}{c}$$

Multiply top and bottom by $\sqrt{a}+b$

Multiply out the numerator (the cross-terms will cancel)

Cancel out a common term on top and bottom

Plug in the limit

Rationalizing the Numerator

Example:

$$\lim_{x \to 5} \frac{\sqrt{x^2 + 144} - 13}{x - 5}$$

Rationalizing the Numerator

Example:

$$\lim_{x \to 16} \frac{4 - \sqrt{x}}{16x - x^2}$$

Piecewise functions

For piecewise functions, calculate the limit by plugging in for x, but you need to decide which equation to plug it into.

$$f(x) = \begin{cases} x^3 - x + 10, & x < -2 \\ 2x - 2 & x \geq -2 \end{cases}$$

Calculate:

$$\lim_{x \to -2^-} f(x)$$

$$\lim_{x \to -2^+} f(x)$$

Example

For the following function, calculate the limit as $x \to 1^-$, $x \to 1^+$, and $x \to 1$.

$$f(x) = \begin{cases} x^3 + x - 2, & x < 1 \\ 2, & x = 1 \\ e^{x-1} - 1, & x > 1 \end{cases}$$

Example

For the following function, calculate the limit as $x \to -2^-$, $x \to -2^+$, and $x \to -2$.

$$f(x) = \begin{cases} x^2 + 5x, & x < -2 \\ 0, & x = -2 \\ \ln(x+3), & x > 2 \end{cases}$$

Summary

When calculating limits, follow these steps:

Can I plug in the limit and get a simple answer (not $\frac{0}{0}$ or a piecewise function)?

Plug it in and get the answer.

Is the function a fraction where I can factor then cancel or rationalize the numerator?

Factor the numerator and cancel, plug it in to get answer.

Is the function piecewise?

Calculate the left and right limits separately.
If they match, this is the answer.
If they do not match, the limit does not exist.

One more problem

Find the limit

$$\lim_{x \to 0} \frac{x}{\sqrt{1+3x}-1}$$