Transformations of Functions and Exponential Functions

January 24, 2017

Review of Section 1.2

Reminder: Week-in-Review, Help Sessions, Office Hours

Mathematical Models

Linear Regression

Function classes

Review of Section 1.2

Alice's parents recorded her height every 3 years when she was a child. Find the linear regression. Estimate her height at age 8.

Age (years)	Height (in)
3	36
6	42
9	48
12	60

Review of Section 1.2

Classify the function as polynomial, power, rational, algebraic, trigonometric, exponential, or logarithmic.

Review of Section 1.2

Classify the function as polynomial, power, rational, algebraic, trigonometric, exponential, or logarithmic.

Outline of Section 1.3 (New Functions from Old)

Vertical and horizontal shifts

Vertical and horizontal stretching

Composition

Commonly seen classes of functions

Horizontal shifts

To shift the graph of $y = f(x)$ to the left by a units, use

$$y = f(x - a)$$

Horizontal shifts

To shift the graph of $y = f(x)$ to the right by a units, use

$$y = f(x + a)$$

Vertical shifts

To shift the graph of $y = f(x)$ up by a units, use

$$y = f(x) + a$$

Vertical shifts

To shift the graph of $y = f(x)$ down by a units, use

$$y = f(x) - a$$

Question

Why is the sign reversed for horizontal shifts?

Horizontal stretching

To stretch the graph of $y = f(x)$ out horizontally by a factor c, use

$$y = f(x/c)$$

Vertical stretching

To stretch the graph of $y = f(x)$ out vertically by a factor c, use

$$y = cf(x)$$

Question

Why do you divide by c for horizontal stretching?

Reflection across the y-axis

To reflect the graph of $y = f(x)$ across the y axis, use

$$y = f(-x)$$

Reflection across the x-axis

To reflect the graph of $y = f(x)$ across the x axis, use

$$y = -f(x)$$

Important note

Remember: To reflect across the **x**-axis, multiply the **y**-value by -1. To reflect across the **y**-axis, multiply the **x**-value by -1

Summary

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c

$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c

$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c

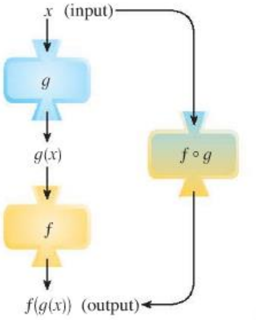
$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

Function composition

The composition $(f \circ g)(x) = f(g(x))$ means plugging $g(x)$ into $f(x)$

Think of functions like machines



Function composition

Example:

$$f(x) = \sin(x), \qquad g(x) = x^2 + \frac{1}{x}$$

Function composition

Example:

$$f(x) = \sqrt{x+1}, \qquad g(x) = \frac{x+1}{x-1}, \quad h(x) = x^9$$

Function composition

Example:

$$f(x) = \sin^2(x+2)$$

Find a functions g, h, k such that $f = g \circ h \circ k$.

Function composition

Example:

$$f(x) = -2x^2 + 2x - 1, \qquad g(x) = -2x + 1$$

Find a function $h(x)$ such that $(g \circ h)(x) = f(x)$

Function composition

Example:

$$f(x) = x^2 - 4x + 1, \qquad g(x) = -x + 2$$

Find a function $h(x)$ such that $(h \circ g)(x) = f(x)$

Function composition

Example:

$$f(x) = 3x^2 + 6x - 2, \qquad g(x) = 3x + 1$$

Find a function $h(x)$ such that $(g \circ h)(x) = f(x)$

Domain review

Find the domain of

$$f(x) = \log(2 - \sqrt{x+1})$$

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Outline of Section 1.5

Exponential functions

Rules for manipulating exponential functions

Applications: Exponential growth and decay

Compound interest

Rate of growth of exponential functions

Exponential functions revisited

Recall

An **exponential function** is a function of the form

$$f(x) = a^x,$$

where a is a postive constant.

Exponential rules

Exponential rules

If a and b are positive numbers then,

$$a^{x+y} = a^x a^y \qquad a^{x-y} = \frac{a^x}{a^y} \qquad (a^x)^y = a^{xy} \qquad (ab)^x = a^x b^x$$

Exponential rules

3^0

$81^{1/2}$

$8^{4/3}$

$4^{-1/2}$

$5^3 \cdot 5^{-5}$

Exponential rules

$(3^2)^3$

$7^2 \cdot 4^2$

$(2+3)^2$

$\sqrt{4+9}$

Exponential rules

Simplify the expression

$$(4x^6)^{-1/2} \cdot x^2$$

Exponential decay

The half-life of Carbon-14 is 5730 years. If a sample initially contains 5mg of Carbon-14 at time $t = 0$, calculate the amount of Carbon-14 in the sample at an arbitrary time $t \geq 0$.

Exponential growth

Suppose that a population of bacteria doubles every two hours, and the initial population of a sample is 300. Find the population of the sample 5 hours later.

Exponential growth

A country has an annual population growth rate of 3%. Assuming exponential growth, how many years will it take for the population to double?

Converting to base e

Convert the equation $P = 121(0.89)^t$ to the form $P = P_0 e^{kt}$.

Compound Interest

If P dollars are invested at an annual rate r compounded n times per year, then the amount accumulated after t years is

$$A(t) = P(1 + \frac{r}{n})^{nt}$$

Continuously compounded Interest

If P dollars are invested at an annual rate r compounded continuously, then the amount accumulated after t years is

$$A(t) = Pe^{rt}$$

Rate of growth of exponential functions

Out of all the function classes we've discussed so far, exponential functions of positive base grow the fastest.