Final Exam Review

What to study on your own?

- Previous exams
- Area under a curve (Slides from a week ago)
- Average value of functions (Slides from Tuesday)

Intro

Today, I'm going to walk through some sample problems from old sections to help you remember what we did earlier in the semester

Inverse Functions

Find the inverse function for

$$f(x) = \ln(x-1) - \ln(x+1) + 5$$

Evaluating Limits

Evaluate the limit

$$\lim_{x \to 2} \frac{\sqrt{5 - x^2} - 1}{x - 2}$$

Linear models

Henry believes that more expensive clothes last so much longer than less expensive clothes that buying more expensive clothes is actually cheaper in the long run. He buys two pairs of pants, one costs 15 dollars and one costs 60 dollars. The first lasts 5 years, and the second lasts 8 years.

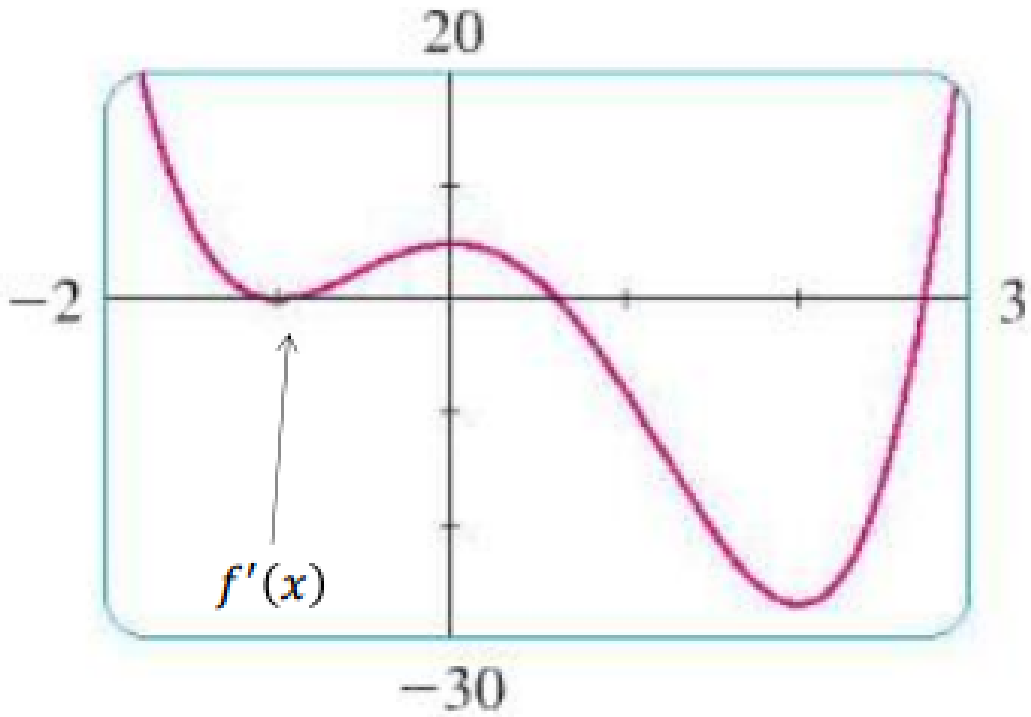
- Create a linear model using these two data points where x is the price in dollars and $f(x)$ is the life-time of the pants.
- Is Henry's belief supported by the model?

Continuity

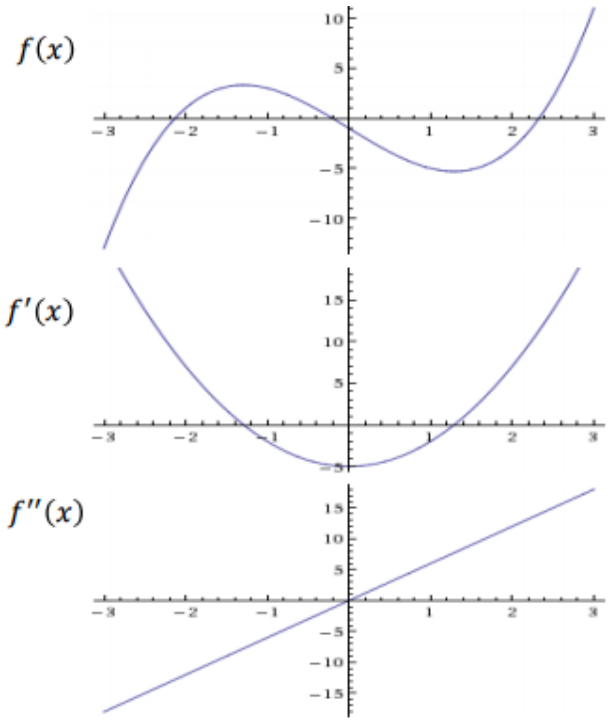
What are the 3 criteria for a function to be continuous at $x = a$?

Information from graphs

Below is the graph of $f'(x)$. Find the local maxes/mins of $f(x)$ and the intervals where $f(x)$ is concave up/concave down.



Concavity and the graph



Slope Concavity

Value
Slope

Value Value

Tangent lines

Calculate the tangent line to the graph at $x = \frac{1}{4}$.

$$f(x) = \cos(\pi x)$$

Definition of the derivative

Calculate the derivative of $f(x)$ at $x = 3$ using the definition of the derivative.

$$f(x) = \sqrt{7 - x}$$

Combining Derivative Rules

Calculate the derivative of $f(x)$.

$$f(x) = \sin(\ln(e^x + 7x^2 - 1))$$

Acceleration and Velocity

You toss a coin to determine who pays for your recent trip to Chipotle. The height of the coin in inches at time t seconds after being tossed and before hitting the ground is given by

$$h(t) = -3t^2 + 10t + 48$$

- What is the coin's velocity when it hits the ground (position equals 0)?
- At what height does the veloicty equal 0?

Antiderivative

Find the exact antiderivative of $f'(x)$ given that $f(1) = 0$.

$$f'(x) = \frac{3}{x} + \sin(\pi x)$$

Fundamental theorem of calculus

Use the fundamental theorem of calculus to calculate

$$\frac{d}{dx}\int_0^{x^2} \ln(\tan(t+t^2))\,dt$$

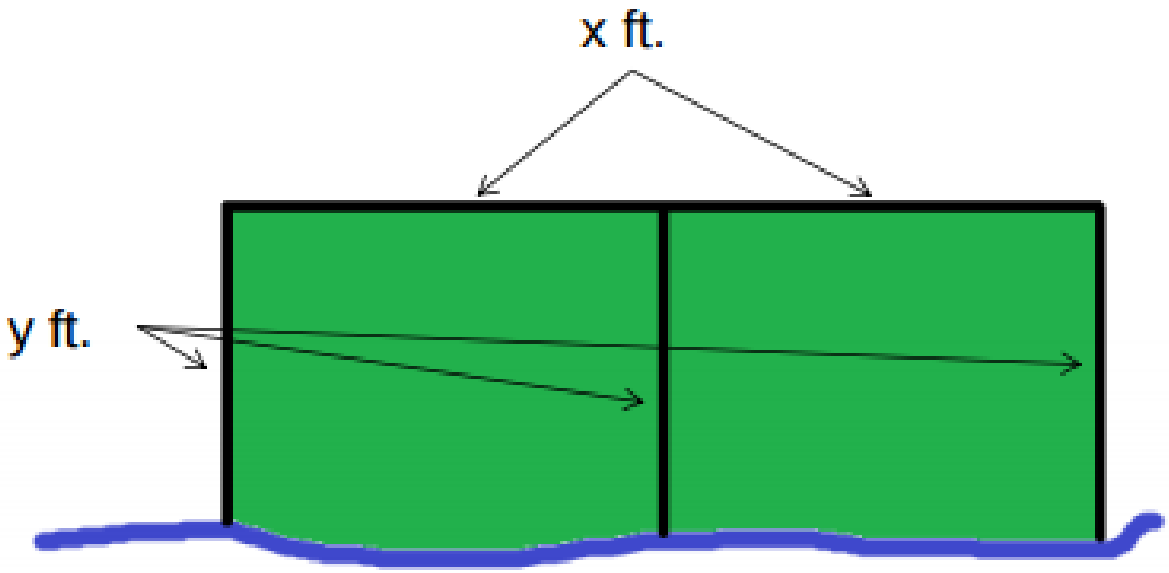
Absolute extrema

Find the absolute min and max on $[0, 4]$ for

$$f(x) = -x^2 + 2x + 15$$

Optimization problems

Warning: there are many different types of these problems. Be sure to study a variety of them.
A farmer builds a field with a fence as shown below. He has 60 ft. of fence availabel. What is the maximum area he can enclose?



Riemann sums

Use a right endpoint estimate with $n = 4$ to estimate the integral

$$\int_1^3 \sin\left(\frac{x}{2}\right)\, dx$$

Substitution

Evaluate the indefinite integral.

$$\int \frac{\sin(\ln(x))}{x}\, dx$$

Definite integrals

Evaluate the definite integral.

$$\int_3^5 (1-2x)^2\,dx$$

Shape of graph

For the function $f(x) = 2x^3 - 9x^2 - 60x + 120$,

- Find the intervals where $f(x)$ is increasing/decreasing
- Find the local maxes/mins
- Find the inflection points
- Find the intervals where $f(x)$ is concave up and concave down

More acceleration and velocity

You throw a penny downwards off the Rudder Tower with a speed of 8 m/s. Rudder tower is 150 m tall and gravity is 10 m/s^2 (ignore air resistance).

- After how many seconds is your penny going 48 m/s?
- How fast is the penny going when it hits the ground? Is this dangerous if it hits someone?