

Product and Quotient Rules (Section 3.2) and Derivatives of Trigonometric Functions (Section 3.3)

Product Rule

When two functions are multiplied together, we use the **product rule** to take the derivative of the product.

If

$$f(x) = u(x) \cdot v(x),$$

then

$$f'(x) = u'(x)v(x) + u(x)v'(x)$$

Example

Compute the derivative of $f(x)$.

$$f(x) = e^x x^2$$

Example

Compute the derivative of $f(x)$.

$$f(x) = (x^2 + 1)(x^3 - 2x - 2)$$

Example

Compute the derivative of $f(x)$.

$$f(x) = (x - e^x) \left(\sqrt{x} - \frac{1}{x^5} \right)$$

Combining multiple rules

When more than two functions are multiplied together, we have to apply the product rule multiple times.

Example

Compute the derivative of $f(x)$.

$$f(x) = (x^2 - x)(x^4 - 3x^2 - 2)(5x - 3)$$

Quotient rule

If $f(x)$ is a fraction, then we use the **quotient rule** to take the derivative.

If

$$f(x) = \frac{u(x)}{v(x)},$$

then

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

Quotient rule

Abbreviated:

If

$$f(x) = \frac{u}{v},$$

then

$$f'(x) = \frac{u'v - uv'}{v^2}$$

Example

Compute the derivative of $f(x)$.

$$f(x) = \frac{3x^2}{x^3 - 1}$$

Example

Compute the derivative of $f(x)$.

$$f(x) = \frac{e^x + x^2}{5x^3 + x}$$

Example

Compute the derivative of $f(x)$.

$$f(x) = \frac{3x - 5e^x}{2x - x^3}$$

Example

Compute the derivative of $f(x)$.

$$f(x) = \frac{x^3 - 2x^2 + 1}{x^3}$$

Example

Combine derivative rules to find the derivative of f .

$$f(x) = \frac{-2e^x x^4}{x^3 - 3}$$

Example

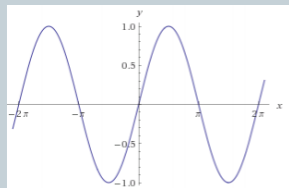
Find the derivative of f .

$$f(x) = \frac{1 - \left(\frac{1}{\sqrt{x}} - x\right) e^x}{1 + \sqrt{x}}$$

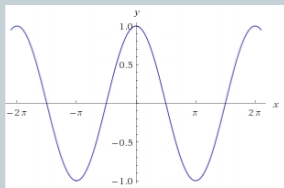
Trig functions

Recall the basic trig functions:

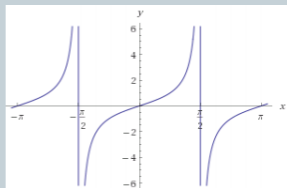
$$f(x) = \sin(x)$$



$$f(x) = \cos(x)$$



$$f(x) = \tan(x)$$



Basic trig derivatives (Memorize!)

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

$$(\tan(x))' = (\sec(x))^2 = \sec^2(x)$$

3 more trig derivatives (Memorize!)

$$(\csc(x))' = -\csc(x) \cot(x)$$

$$(\sec(x))' = \sec(x) \tan(x)$$

$$(\cot(x))' = -(\csc(x))^2 = -\csc^2(x)$$

Example derivations

Find the derivative.

$$f(x) = \csc(x)$$

$$f(x) = \tan(x)$$

Example

Find the derivative:

$$f(x) = \sin(x) \cos(x)$$

Example

Find the derivative:

$$f(x) = \frac{\tan(x)}{\sec(x)}$$

Example

Find the derivative:

$$f(x) = \frac{x^2 \cos(x)}{e^x + \cot(x)}$$

Spring Example

When a spring is pulled and released, the position function for its free endpoint is given by

$$p(t) = -5 \cos(t) + 5$$

How fast is the spring moving after 1 second?

When is the spring moving the fastest?