What does f' say about the graph of f? (Section 2.8) and Derivatives of Polynomials and Exponential Functions (Section 3.1)

Introduction

The derivative of a function $f(x)$ gives a lot of information about the graph of $f(x)$. It also tells you about the maxes and mins of $f(x)$.

Overview

Derivative and Shape of Graph

Maxes/Mins

Second Derivative and Shape of Graph

Shortcut rules for derivatives

Recall

The derivative gives the slope of a graph.

- Plugging x-values into $f(x)$ gives the y-values of the graph of $f(x)$
- Plugging x values into $f'(x)$ gives the slope of the graph of $f(x)$
- The value on the $f'(x)$ graph corresponds to the slope on the $f(x)$ graph

Examples

Extrema

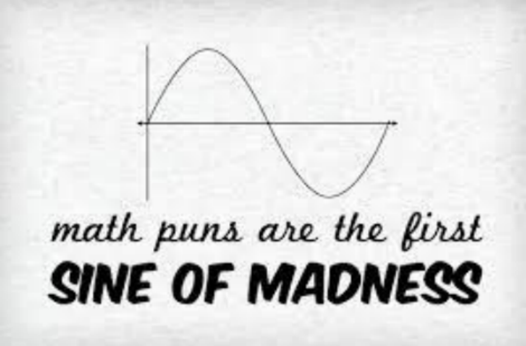
Local extrema on a graph are peaks and valleys

The peaks are called local **maxima** (singular: maximum)

The valleys (low points) are local **minima** (singular: minimum)

Examples

Math Joke



More precise definitions (Important to remember)

When we say "local max/min" we mean a point (x, y).

The **value** of the max/min is the y-value.

Second derivatives

Remember: The **second derivative** of a function $f(x)$ is found by taking the derivative twice.

In other words, find $f'(x)$ then do $(f'(x))'$

The second derivative tells us how “curved” the graph is.

Concavity

The sign of $f''(x)$ tells us about which way the graph of $f(x)$ curves.

If $f''(x) > 0$ on an interval (a, b), then the function $f(x)$ is **concave up** on that interval.

Concave up = "smiley face"

If $f''(x) < 0$ on an interval (a, b), then the function $f(x)$ is **concave down** on that interval.

Concave down = "frowny face"

Points where the curve changes concavity are called **inflection points**

Cubic function example (Concavity)

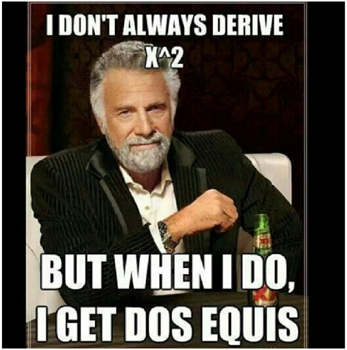
Cubic function example (all three graphs)

Power rule

The rule for taking the derivative of the function $f(x) = x^n$, where n is any number is:

$$f'(x) = nx^{n-1}$$

Math Joke



Examples

$$f(x) = x$$

$$f(x) = x^{11}$$

$$f(x) = \sqrt{x}$$

$$f(x) = \frac{1}{x^2}$$

Constant rule

The derivative of any number (i.e., the variable x does not appear in the term) is 0.

Constant multiple rule

If a function has a number multiplied out front, then we can ignore that number while taking the derivative.

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$$

Addition/subtraction rule

If two pieces of a function are added/substracted to each other, we may calculate the derivative of each piece separately.

$$(f(x) + g(x))' = f'(x) + g'(x)$$

Note: Does NOT work when multiplied or divided!

Example

Calculate the derivative of the following function:

$$f(x) = 3x^{2/3} - \frac{1}{x^2} + 5 \cdot 3^6$$

Example

Calculate the equation of the tangent line to $f(x)$ at $x = 1$.

$$f(x) = x^3 - \frac{2}{x^2}$$

Example

Calculate the equation of the tangent line to $f(x)$ at $x = 4$.

$$f(x) = \sqrt{x} - \frac{1}{x} + 2$$

Rule for e^x

The derivative of $f(x) = e^x$ is easy!

$$\frac{d}{dx} e^x = e^x$$

Note: This rule changes if there is anything else in the exponent, e.g. e^{2x}
(We'll talk about how to differentiate this next time.)

Examples