The Derivative as a Function (Section 2.7)

Introduction

The main point of this lecture is to plug the variable x into the derivative, instead of a constant a.

This means we are thinking of the derivative as a function of x, instead of just a number.

Overview

Finish derivative examples from last time

Derivative as a function

Differentiability

Derivative as Number

Last lecture we saw the formula for the derivative at $x = a$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

We plugged in a number for a and got a number.

Derivative as Function

Today we replace a with the variable x:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The answer now is a function of x. The h will disappear after evaluating the limit.

Derivative as Function

Connection:
If we get $f'(x)$ as a function, we can plug in any number a to get the same answer as last time.

Derivative as Function

In general, when we say "calculate the derivative of $f(x)$" we mean calculate its derivative as a function.

Example

Find the derivative of

$$f(x) = 2x^2 - 3$$

Comparing the graphs from the last example

Example

Find the derivative of

$$f(x) = \frac{1}{1-x} + 2$$

Notation

Usually we write

$$f'(x) = derivative$$

Sometimes we write

$$\frac{d}{dx}f(x) = \frac{df}{dx} = \frac{dy}{dx} = derivative$$

Differentiability

A function $f(x)$ is **differentiable** at $x = a$ if the derivative exists at $x = a$.
Ways to check:
The limits match:

$$\lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}.$$

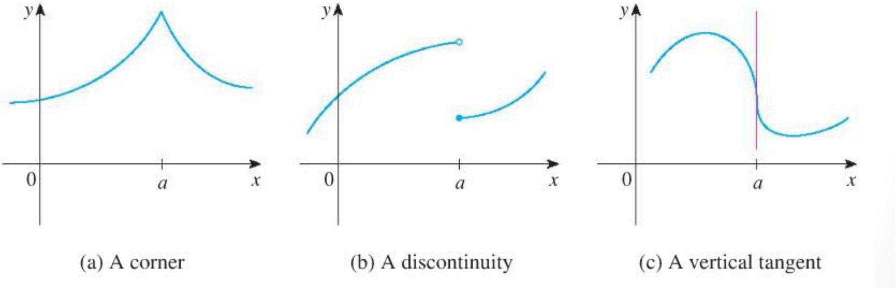
OR
Calculate the derivative $f'(x)$ and check that you can plug in $x = a$.

Not Differentiable

Example of a function which is not differentiable at $x = 0$:

$$f(x) = |x|$$

Not Differentiable



Differentiabity requires Continuity

If $f(x)$ is differentiable at $x = a$, the $f(x)$ must be continuous at $x = a$.
Contrapositive:

Higher derivatives

Doing the derivative twice or more:

$$f''(x) = \text{ second derivative}$$

$$f'''(x) = \text{ third derivative}$$

The second derivative of position wrt time is the instantaneous acceleration.
The third derivative of position wrt time is the “jerk”.