Limits Involving Infinity (Section 2.5)

Feb 9, 2017
9:35 - 10:50 AM

Outline

Infinity in the answer

Infinity in the problem

Infinity in the answer

Plug a number into an expression and get infinity out

Infinity in the answer

When we write

$$\lim_{x \to a} f(x) = \infty$$

or

$$\lim_{x \to a} f(x) = -\infty,$$

we are saying that the limit **does not exist**, and we are **describing** the way it does not exist.

Example

$$f(x) = \frac{1}{x}$$

You can a calculator to see the behavior near 0.

Example

$$f(x) = \frac{1}{(x-1)^2}$$

Example

$$f(x) = \frac{x}{x+5}$$

Example

$$f(x) = \frac{(x+5)(x-3)}{(x+2)(x-5)}$$

Procedure for determining the sign of an infinite limit

Plug the limit x-value into all non-zero factors

Use a number line to determine signs of zero factors (0^+ and 0^-)

Count the number of negatives

Even -> $+\infty$, odd -> $-\infty$

Example

$$f(x) = \frac{(x+4)(x-8)}{(x+1)(x-6)}$$

Vertical asymptotes

Whenever there is an infinite discontinuity in a graph at $x = a$, we say that there is a vertical asymptote at $x = a$.

Log limit

Note:

$$\lim_{x \to 0^+} \ln(x) = -\infty$$

Infinity in the problem

$$\lim_{x \to \infty} \frac{x+1}{x-2}$$

You can use a calculator to understand the behavior as $x \to \infty$

Procedure for limits of fractions as $x \to \pm\infty$

Identify the highest power term in the fraction

Multiply the top and bottom with $\frac{1}{x^n}$ for that term

Plug in ∞, everything with ∞ in the denominator goes away

What's left is the answer

Horizontal asymptotes

If

$$\lim_{x \to \infty} f(x) = c$$

for some number c or

$$\lim_{x \to -\infty} f(x) = c$$

for some number c, then we say $f(x)$ has a **horizontal asymptote** at $y = c$.

Example

$$\lim_{x \to -\infty} \frac{x^2 - 4x + 1}{3x^2 - 5}$$

Example

$$\lim_{x \to \infty} \frac{5x^2 - 4x + 1}{2x - 5}$$

Example

$$\lim_{x \to \infty} \frac{x^2 - 3x + 4}{x^3 + 1}$$

Example

$$\lim_{x \to -\infty} \frac{10x^3 - 4x + 1}{4x^3 + 8x^2 + x - 5}$$

Fractions involving exponential functions

$$\lim_{x \to \infty} e^x = \infty$$

and

$$\lim_{x \to -\infty} e^x = 0.$$

The function e^x grows faster than any polynomial as $x \to \infty$.

Example

$$\lim_{x \to \infty} \frac{3e^x - 2x + 1}{2e^x + x - 5}$$

Example

$$\lim_{x \to -\infty} \frac{3e^x - 2x + 1}{2e^x + x - 5}$$

Summary

We learned two different ways of dealing with limits with infinities, depending on whether the infinities appear in the question or the answer.