Approximating Slopes of Tangent Lines
Introduction to Limits

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9:35 - 10:50 AM

Outline

Review of inverse functions and logs

Tangent lines

Average velocity

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Limits

Tangent lines

The tangent line to a graph $y = f(x)$ at a point x_0 is the line that barely touches the graph at $(x_0, f(x_0))$.

Average velocity

Given a function $f(t)$ describing the position of an object at time t, its average velocity between times t_0 and t_1 is

$$v_{ave} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

Instantaneous velocity

Given a function $f(t)$ describing the position of an object at time t, its instantaneous velocity at time t_0 is the slope of the tangent line at $t = t_0$.

Estimating slopes of tangent lines

Two ways to estimate the slope of the tangent line at $x = x_0$:

- Secant lines approaching x_0 from the right
- Secant lines approaching x_0 from the left

Estimating slopes

Estimate the slope of the tangent line at $t = 1$ using each of the 4 other data points.

t (sec)	dist (m)
5	22
2	1
1.1	-1.79
1.01	-1.9799
1	2

Estimating slopes

The distance an airplane which is taking off has travelled is given by the formula

$$D(t) = 3t^2 - t + 5$$

Estimate the velocity of the airplane at $t = 3$ by using time intervals starting at $t = 3$ and lasting 1, .5, .1, and .01 seconds.

Estimating slopes

An electron moves along a wire in an AC circuit according to the formula

$$x(t) = 5\sin\left(\frac{\pi t}{2}\right)$$

Estimate the velocity of the electron when $t =$ by using the time intervals $[2, 4]$, $[3, 4]$, $[3.5, 4]$, $[3.9, 4]$, and $[3.99, 4]$.

Limits

To take the limit of a function $y = f(x)$ at the point $x = a$, we look at the y-values as the x-values get closer to a.

Example

Let

$$f(x) = x^2 + 2$$

To calculate the limit at $x = 2$, we compute the following values:

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	4.61	4.9601	4.996001	5.004001	5.0401	5.41

We say the limit as x approaches 2 equals 5, or $\lim_{x\to 2} f(x) = 5$.

Remark

Limits are important when the graph displays unusual behavior around the point in question. On the previous example, we could have just plugged $x = 2$ into the function.

Removable discontinuity example

Let

$$f(x) = \frac{x^2 - 9}{x - 3}$$

To calculate the limit at $x = 3$, we compute the following values:

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	5.9	5.99	5.999	6.001	6.01	6.1

So $\lim_{x \to 3} f(x) = 6$.

Limits of piecewise functions

Let

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2x + 5, & x \geq 0 \end{cases}$$

Left and right-handed limits

In this case, we write

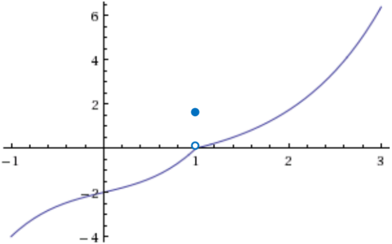
$$\lim_{x \to 0^-} f(x) =$$

$$\lim_{x \to 0^+} f(x) =$$

The actual (two-sided) limit $\lim_{x \to 0} f(x)$ only exists if the left and right-handed limits are equal. They are not, so the limit DNE (does not exist)

Example

$$f(x) = \begin{cases} x^3 + x - 2, & \text{if } x < 1 \\ 2, & \text{if } x = 1 \\ e^{x-1} - 1, & \text{if } x > 1 \end{cases}$$



Example