

# Inverse Functions and Logarithms

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# Outline of Section 1.6

One-to-one functions

Horizontal line test

Inverse functions

Logarithms

# Motivation

To solve equations involving nontrivial functions, we need their inverse functions.

One-to-one functions have well-defined inverses

## One-to-One (1-1) functions

A **one-to-one** (or injective) function is a function for which there is a unique  $x$ -value for every  $y$ -value.

## Horizontal line test

A function is one-to-one if and only if no horizontal line intersects its graph more than once.

# Inverse functions

A one-to-one function  $f$  has an inverse function, written

$$f^{-1}(x)$$

Defining rule: If  $f(x) = y$ , then  $f^{-1}(y) = x$ .

# Graphing inverse functions

# Inverse functions

If a one-to-one function  $f(x)$  has domain  $A$  and range  $B$ , then its inverse function  $f^{-1}(x)$  has domain  $B$  and range  $A$ .



Don't confuse  $f^{-1}(x)$  with  $(f(x))^{-1}$

Don't confuse

$$f^{-1}(x)$$

with the fraction

$$(f(x))^{-1} = \frac{1}{f(x)}.$$

## Example

Let  $f(x) = \sqrt{x}$ .

## Example

$x$	$f(x)$	$x$	$g(x)$
1	12	1	-3
2	7	2	-1
5	3	3	1
7	-1	4	3

1. Find  $f^{-1}(3)$  and  $g^{-1}(-1)$
2. Find  $f^{-1}(g(2))$
3. Find  $g^{-1}(f^{-1}(12))$

# Calculating inverses

To calculate the inverse of the function  $f(x)$ :

Write the function as  $y = f(x)$

Swap the  $x$  and  $y$ .

Solve for  $y$ .

# Using inverse functions

The inverse function *undoes* a function

$$f(f^{-1}(x)) = x$$

and

$$f^{-1}(f(x)) = x$$

# Logarithms

The inverse of the exponential function is called the **logarithm**.

If  $f(x) = a^x$ , then

$$f^{-1}(x) = \log_a(x)$$

The number  $a$  is called the **base** of the logarithm.

# Logarithms

This means logarithms and exponentials undo each other:

$$\log_a(a^x) = x$$

and

$$a^{\log_a(x)} = x$$

## Example

Logarithms grow very slowly. Example: solve for  $\log_{10}(x) = 8$ .



# Logarithm rules

$$\log_a(a)$$

$$\log_2(5 \cdot 6)$$

$$\log_5\left(\frac{7}{10}\right)$$

$$\log_{10}(3^7)$$

## Example

Apply the logarithm rules to simplify

$$\log_2(10) + \log_2(14) - \log_2(35)$$

# Natural logarithm

The inverse of the natural exponential function is the natural logarithm.

$$f(x) = e^x$$

$$f^{-1}(x) = \ln(x) = \log_e(x) = \log_{2.718...}(x)$$

# Change of base formula

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

## Example

Find the domain and inverse of

$$f(x) = \sqrt{e^{2x} - 1}$$

## Example

Find the domain and inverse of

$$f(x) = \ln(\ln(x) - 1)$$

## Example

Find the domain and inverse of

$$f(x) = \sqrt{\ln(x + 5)}$$